Problem Sheet 4 – submit by August 31st

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Question 1: King James is increasingly worried about witchcraft sweeping the kingdom: the latest reports reveal that 1% of his subjects are witches. He summons his royal doctors and they tell him about a new scientific method to test whether a person is a witch. 90% of witches float when hurled into a river. However, the smallprint notes that the test has a 9.6% false positive rate. King James does not read the smallprint and orders that everyone in his kingdom be tested for witchcraft immediately.

a) James declares that anyone who tests positive has a 90% chance of being a witch. Is he right? If not, why not? Which fallacy does this connect to?

b) Assuming the above information is correct, suppose that we find someone floats in water. What's the probability that the person is a witch?

c) The court statistician decides to have a word with King James, and explains the mistake he is making. So King James proposes a new idea. Rather than a single test, he decides to throw people into a river twice. Anyone who floats **both** times is likely to be a witch. Is he right?

Question 2: King James has become convinced that leeches are a cure for the Black Plague. In reality, the claim "leeches cure the Black Plague" is false. But King James does not know this. He asks his top team of royal physicians to each conduct a scientific trial to test whether leeches cure the Black Plague. The team will declare a statistically significant "discovery" that leeches cure the Black Plague if they can attain a p-value of < 0.05.

a) What is the null hypothesis in this experiment?

b) What does it mean to attain a p-value of < 0.05? You may want to include a diagram in your explanation.

c) How likely is it that the physicians will attain a p-value of < 0.05?

d) In fact, the royal physicians find a p-value of 0.1. Can they claim a discovery?

e) Frustrated with the results, King James orders 10 more trials to be conducted. Assuming that each trial is independent, what is the probability that at least one trial will attain a statistically significant (at p-value < 0.05)?

f) King James orders that only trials with a statistically significant finding should be published. Does that seem like a good idea? Why or why not?

Question 3: Here's a claim from a well-known *Statistics in Biology* textbook.

to find out how likely that null hypothesis is to be true. If we decide to reject the null hypothesis we must accept the alternative, more interesting hypothesis (H_1) that: 'the weights of bulbs for the two cultivars are different'.

P-values

The *P*-value is the bottom line of most statistical tests. It is simply the probability that the hypothesis being tested is true. So if a *P*-value is given as 0.06, that indicates that the hypothesis has a 6% chance of being true. In biology it is usual to take a value of 0.05 or 5% as the critical level for the rejection of a hypothesis. This means that providing a hypothesis has a less than one in 20 chance of being true, we reject it. As it is the null hypothesis that is nearly always being tested we are always looking for low *P*-values to reject this hypothesis and accept the more interesting alternative hypothesis.

Clearly the smaller the *P*-value the more confident we can be in the conclusions drawn from it. A *P*-value of 0.0001 indicates that the chance of the hypothesis being tested being true is one in 10 000 and this is much more convincing than a marginal $P = 0.049$.

P-values and the types of errors that are implicitly accepted by their use are considered further in Chapter 4.

Sampling

Is the textbook right? If not, explain precisely what the textbook has gotten wrong.

Question 4: King James has summoned a jury to determine whether Prince Philip is a traitor. There are three jurors.

a) Let's assume that each juror is independent and unbiased, and comes to the right judgement 60% of the time. What's the probability that a majority of the jurors come to the correct judgement? (Be careful to count all the possibilities!)

b) Now suppose there are 5 jurors instead of three. What's the probability that a majority of the jurors come to the correct judgement? (Be careful to count every configuration!)

c) In the limit that there are infinitely many jurors, what does the probability that a majority make the right decision approach?

d) Are the assumptions realistic?